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A quantitative dynamical systems approach to differential learning: self-organization principle and order parameter equations

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Abstract Differential learning is a learning concept that assists subjects to find individual optimal performance patterns for given complex motor skills. To this end, training is provided in terms of noisy training sessions that feature a large variety of between-exercises differences. In several previous experimental studies it has been shown that performance improvement due to differential learning is higher than due to traditional learning and performance improvement due to differential learning occurs even during post-training periods. In this study we develop a quantitative dynamical systems approach to differential learning. Accordingly, differential learning is regarded as a self-organized process that results in the emergence of subject- and context-dependent attractors. These attractors emerge due to noise-induced bifurcations involving order parameters in terms of learning rates. In contrast, traditional learning is regarded as an externally driven process that results in the emergence of environmentally specified attractors. Performance improvement during post-training periods is explained as an hysteresis effect. An order parameter equation for differential learning involving a fourth-order polynomial potential is discussed explicitly. New predictions concerning the relationship between traditional and differential learning are derived.

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1 Introduction

Complex motor skills are used in every-day activities and are of particular importance in sport sciences. Well studied examples of complex motor skills are drumming (Peper et al. 1995; Peper and Beek 1998), juggling (Beek and Turvey 1992; Huys et al. 2004), ball bouncing (Sternad et al. 2001), and pedalo driving (Haken 1996) as well as playing the piano, handwriting and drawing (Shea et al. 1993; Schmidt and Lee 1999). Complex motor skills involve many degrees of freedom related to the large number of the muscular-skeletal components that are inevitably involved in any motor performance. Successful performance requires a high degree of coordination among these degrees of freedom. The question arises how can this coordination be established? In literature there is a long debate centered around this question, i.e., the many degrees of freedom problem (Bernstein 1967; Kay 1988). Several explanations have been proposed ranging from open-loop theories and central pattern generators (Henry and Rogers 1960; Keele 1968) to closed-loop theories (Adams 1971), self-organization (Turvey 1990; Haken 1996), and dynamical systems approaches (Kelso 1995; Beek et al. 1995; Jirsa and Kelso 2004). In line with the latter approaches, self-organization binds the degrees of freedom together to synergies. Roughly speaking, the components of a motor control system behave like people that give applause after a concert. In the beginning, the clapping is non-synchronized, but after a while the audience as a whole settles down in a common rhythmic and synchronized clapping pattern (Neda et al. 2000). Consequently, we observe coordination among the individuals due to self-organization.

Mathematically speaking, in the high-dimensional phase space of a motor control system a low-dimensional order parameter dynamics emerges due to self-organization (Haken 1975, 2004). This order parameter dynamics dominates the entire high-dimensional behavior of the motor control system (circular causality). That is, the notion of self-organization provides us with a comprehensive explanation that involves both micro- and macro-levels of a problem at hand. As a by-product we see that the circular causality principle implies that it is sufficient to focus on the low-dimensional order parameter dynamics.

A further advantage of the dynamical systems approach to human movement is that it naturally addresses the issue of motor learning. In this context, we need to realize that the dynamics of an order parameter is described by a dynamical system that involves an attractor (e.g., a fixed point or limit cycle). During learning a motor control system changes such that a new attractor emerges (Schöner and Kelso 1988a, b). That is, while on the neural and muscular-skeletal level learning implies changes of material parameters such as the weights of synaptic connections and the biomechanical properties of muscles and tendons, on the level of the order parameter dynamics these structural changes result effectively in the emergence of a new attractor. For example, it has been shown that during learning of coordinated rhythmic finger movements that exhibit a phase difference of 90 degrees a fixed-point-attractor close to the required 90 degrees emerges (Schöner et al. 1992).

So far, the dynamical systems approach has been successfully applied in the context of learning processes that are centered around the notion of learning by repetition. Accordingly, there is a well-defined movement pattern or prototype pattern that is specified by the environment (due to training instructions) and has to be learned. As mentioned above this movement pattern corresponds to an attractor of an appropriately defined order parameter model. During learning the attractor is environmentally specified. At the end of the learning process the attractor becomes part of the intrinsic set of potential attractors that are available to a subject (Schöner 1989). We will refer to learning processes as traditional if the to-be-learned movement patterns are oriented on fairly narrow target movements that are specified by training instructions and that are mainly characterized by error corrections and repetitions.

Recently, a qualitatively different learning concept for complex motor skills has been developed called differential learning (Schöllhorn 1999a, b, 2000; Schöllhorn et al. 2006). Accordingly, the aim of differential learning is to support a subject in finding his or her individual, context-dependent performance pattern in order to perform a complex motor skill as successful as possible. That is, the a priori defined idealized performance pattern or prototype pattern is replaced by a subject- and context-dependent performance pattern. Here, context-dependent refers to dependencies on internal parameters (e.g., degree of fatigue) and external parameters (e.g., the relative position with respect to an approaching tennis ball). In order to assist subjects in this learning process, differential training is composed of training exercises that vary qualitatively and quantitatively from exercise to exercise. In this sense, subjects are trained by noisy training sequences. The rationale for this training method is that a permanently changing stimulation encourages a subject to realize the variety of between-exercise differences and the variety of potential patterns that can be performed at all. Subsequently, by putting elements of these patterns together in a self-organized fashion, a subject can find his or her optimal performance pattern for a given task in a given internally and externally defined context. From the dynamical systems perspective, we see that differential learning involves attractors that are not fixed but context-dependent and are not environmentally specified (i.e., specified by training instructions) but emerge due to self-organization.

Differential learning studies should not be confused with studies that explore the variability of practice hypothesis of Schmidt's schema theory (van Rossum 1990; Shea and Wulf 2005). According to schema theory, a movement pattern that can be performed on different spatio-temporal and kinematical scales involves a generalized motor program (GMP) and a parameter scaling law. Training variability can support the learning process of both GMP and scaling law. In this context, training variability means that training exercises are elements of the to-be-learned movement class but differ on appropriately defined scales. In contrast the training exercises in differential learning studies do not belong to a particular movement class. In the context of differential learning such a pre-defined or environmentally specified movement class or schema does not exist (see above). For example, students of a differential learning group are usually instructed to perform actively movement errors (e.g., throw a shot put to the left instead to the front). The philosophy of differential learning is: "never practice the right thing in order to play right" (Schöllhorn et al. 2004). Consequently, training variability in differential learning is different from training variability addressed by schema theory. Note that we will return briefly to this issue in the conclusions (see Sect. 3). However, a detailed discussion about similarities and differences between differential learning and other learning paradigms (repetition of the to-be-learned movement pattern or acquisition of a movement schema) is beyond the scope of the present study and can be found in Schöllhorn et al. (2006).

In sum, our study is concerned with learning of complex motor skills that are learned by means of differential learning techniques, see Fig. 1 (left). A dynamical systems approach that applies to differential learning has not been developed so far. The objective of the present study is to provide such a dynamical systems approach to differential learning, see

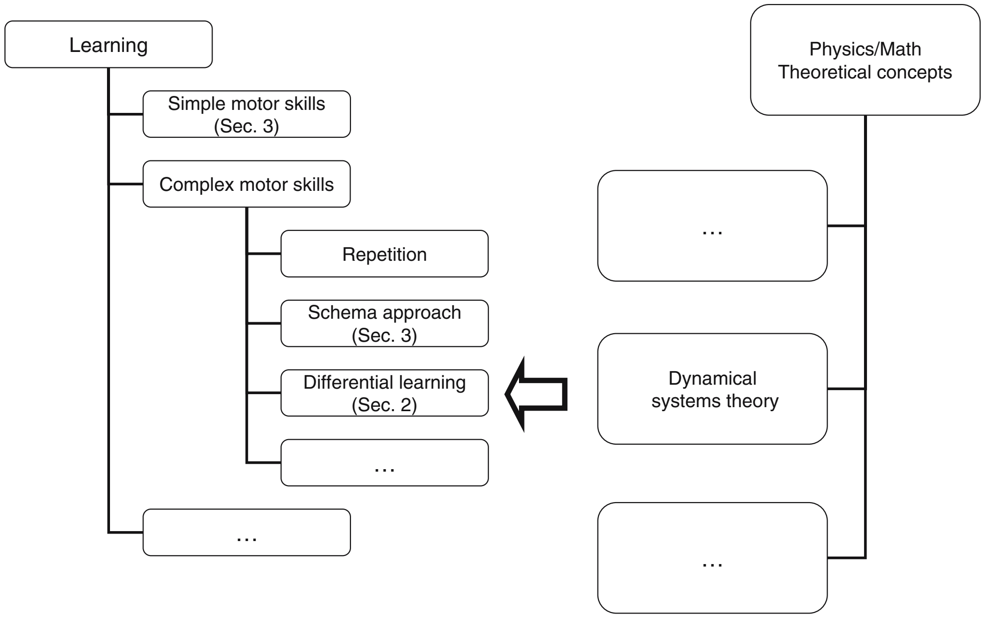

Fig. 1 Illustration of the main goal of the present study: understanding of differential learning processes of complex motor skills by means of dynamical systems theory—as discussed in Sect. 2

Fig. 1 (right). To this end, we will use general theoretical considerations in combination with experimental data that have been collected so far. To this end, in Sect. 2.1 we review an experimental study on shot put that can be regard as a typical example of a differential learning study. In Sect. 2.2 we will derive heuristically from the theory of self-organization a formally-defined order parameter equation for differential learning. In Sect. 2.3 we will discuss an explicit example of an order parameter equation for differential learning. In Sect. 2.4 we will derive new predictions about the relationship between traditional and differential learning that will be used in the Discussion (Sect. 3) to propose future experimentally accessible research directions. In Sect. 3 we will also return to the discussion about the role of variability in schema theory and differential learning. In addition, in Sect. 3 we will address differences and similarities between dynamical systems approach to learning of simple and complex motor skills.

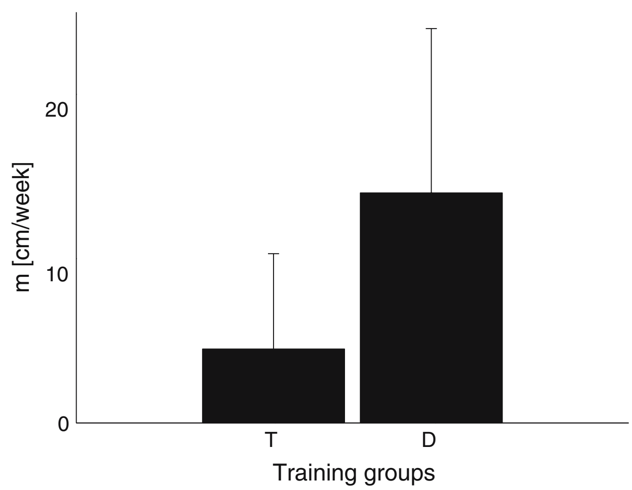

Fig. 2 Improvement per time unit m for the traditional (T) and the differential (D) learning method observed in the shot put study by Beckmann (2003) and Beckmann and Schöllhorn (2003)

2 On a dynamical systems perspective of differential learning

2.1 Experimental studies

In several studies traditional and differential training methods have been compared with each other (Schöllhorn et al. 2006). We will review here a typical example of a differential learning study. The object of this training study was to improve the performance in shot put exercises. Figure 2 shows results from this training study (Beckmann 2003; Beckmann and Schöllhorn 2003). Two groups of students (age 22.1 ± 3.8, 2 × 12 subjects) participated in the study. The first group (T group) was trained with a traditional training method. The second group (D group) was trained by means of the differential learning method. In particular, the exercises for the D group varied with respect to initial and final interlimb angles, the direction of shot impact, the timing of lower and upper limbs, and the way the shot had to be held. The performance improvements of both groups are shown in Fig. 2. After training both groups showed improved performance. That is, on the average both groups were able to throw a longer distance than before training. However, the improvement of the differential training group was significantly higher ($t = -2.707$, $df = 22$, $p = 0.01$).

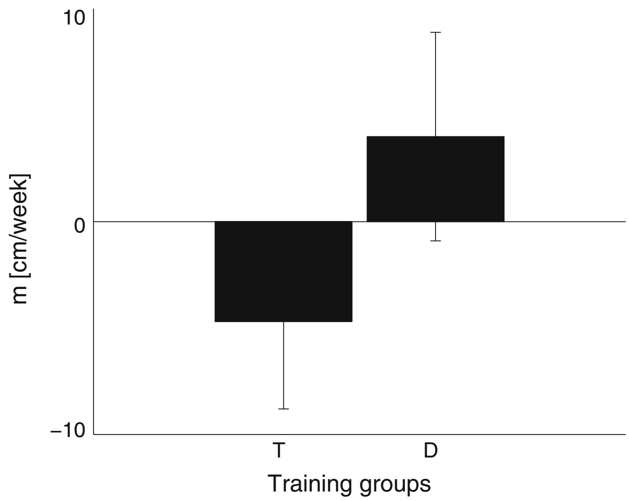

Fig. 3 Improvement per time unit m in post-training periods observed in the shot put study by Beckmann and Schöllhorn (2003)

Table 1 Improvement of throwing range (in cm/week) observed in the shot put study by Beckmann and Schöllhorn (2003)

Groups	Training period	Post-training period
T	4.5 ± 5.8	-4.7 ± 4.1
D	14 ± 10	4.0 ± 4.9

Performance improvement in differential learning groups was significantly better than in traditional learning groups not only in this shot put study but similar results have been found in tennis training (Humpert 2004; Humpert and Schöllhorn 2006) and volleyball (Römer et al. 2003).

In the shot put study, 4 weeks after training a retention test was made with students of both groups. As shown in Fig. 3 the differential learning group improved their performance even during this post-training period. In contrast, performance of the traditionally training group did not improve. This difference was significant ($t = -4.485$, $df = 22$, $p = 0.001$). The improvement per week for both groups during the training and post-training period is reported in Table 1. Note that a similar effect was found in the aforementioned tennis study.

2.2 Self-organization and order parameter equation

The key step in differential learning is to induce a process of self-organization that yields an individual and context-dependent optimal performance pattern for a to-be-learned motor skill. However, self-organization in general emerges due to bifurcations (Haken 1975, 2004). Therefore, our working hypothesis of a self-organized learning process implies that differential learning emerges from a bifurcation. In this context it is plausible to regard the noisiness of the training exercises as control parameter. This control parameter induces at a particular threshold the bifurcation from an unsuccessful random learning behavior to a successful learning process based on self-organization.

Unsupervised learning during post-training periods of differential training—as reviewed in Sect. 2.1—can be explained as an hysteresis effect: if the noisiness of the training exercises is reduced during the post-training period the motor control system is still trapped in the state of self-organized learning. In short, we may distinguish between three stages of learning for subjects who participate in differential training sessions (D-group participants), see Table 2. In the pre-training period there is no coach or instructor. Likewise, there is neither training nor self-organized learning. During training there is a bifurcation to supervised self-organized learning. In the post-training period the coach or instructor is absent again. Nevertheless, there is learning, which corresponds—according to our model—to an on-going unsupervised self-organized learning process.

In order to put these considerations into a quantitative form, we use the performance improvement per time unit (where the time unit can be chosen appropriately) as order parameter m. Note that we may simply refer to m as learning rate. For example, in the shot put experiment students of the D group on average improved their throwing range by about 56 cm in 4 weeks yielding $m = 14$ cm/week. This definition of the order parameter has two advantages. First, the order parameter m thus defined can capture the bifurcation to differential learning in terms of a transition like $m = 0 \rightarrow m > 0$. Second, m can be related to experimental data, see Figs. 2 and 3 and Table 1.

In what follows the training noise (control parameter) will be denoted by Q, where the training noise is a measure for the variety of between-exercise differences. Note that a more instrumental definition of Q is not necessary at that stage of our argumentation. The evolution of the order parameter m with time t is in general determined by an order parameter equation of the form (Kelso 1995; Haken 1996)

$$\frac{\mathrm{d}}{\mathrm{d}t} m(t) = -\frac{1}{\tau_D} \frac{\mathrm{d}V(m)}{\mathrm{d}m}, \qquad (1)$$

Table 2 Stages of learning of D-group participants during the shot put study

	Periods		
	Pre-training	Training	Post-training
Supervision/instructions	No	Yes	No
Self-organized learning	No	No → Yes (bifurcation)	Yes

Table 3 Self-organization in an equilibrium ferromagnetic system (a) and in a nonequilibrium differential learning system (b)

(a) Ferromagnetism		
Control parameter		
Temperature	High	Low
Macroscopic state	Paramagnetism	Ferromagnetism (self-organized state)
Order parameter		
Magnetization	Zero	Larger than zero
(b) Differential learning		
Control parameter		
Variation of differential training	Low	High
Macroscopic state	No learning effect	Differential learning (self-organized state)
Order parameter		
Learning rate	Zero	Larger than zero

where $V(m)$ denotes a potential function. In Eq. (1) we have introduced the parameter $\tau_D > 0$ that characterizes the relaxation time of the order parameter dynamics related to differential learning. For the moment we may absorb τ_D into the potential V or put $\tau_D = 1$ TU, where TU denotes an appropriately chosen time unit. The value of τ_D will become only crucial in Sect. 2.4 below.

The minima of the potential function $V(m)$ describe stable fixed points of the order parameter m. The self-organized state of differential learning is assumed to emerge due to a noise-induced transition (Horsthemke and Lefever 1984). That is, if the control parameter Q is increased beyond a critical threshold Q_c, then minima of V may vanish and new minima may occur such that m bifurcates from $m = 0$ to $m > 0$. In other words, we consider the situation in which for noise levels $Q < Q_c$ the potential $V(m)$ exhibits a minimum at $m = 0$. This minimum becomes a maximum for $Q > Q_c$. In addition, for $Q > Q_c$ the potential is assumed to exhibit a minimum at a finite value $m > 0$. Accordingly, noisy training instructions as given in differential training sessions increase the noise level Q such that it exceeds the threshold Q_c and a self-organized learning process emerges with $m > 0$. In general, m will depend on Q. Let $f(Q)$ denote the function that describes the relationship between Q and m for $Q > Q_c$ if Q is increased (i.e., if we have $Q \uparrow$). Then, the order parameter m as a function of the noise level Q can be described formally by

$$m(Q \uparrow) = \begin{cases} 0 & Q < Q_c \\ f(Q) & Q > Q_c \end{cases} \quad (2)$$

As indicated in Eq. (2), the relationship (2) between m and Q holds only if we consider an increasing control parameter Q. If we decrease the noise level ($Q \downarrow$) a different relationship may hold. In post-training periods we assume that the noise level is below the critical value Q_c. Note that in general Q does not vanish but is finite because during the post-training period every-day motor performance acts as noisy training exercises. We assume that the potential V exhibits still a potential minimum at $m > 0$ for $Q < Q_c$. Consequently, if the noise level drops below the critical value Q_c, a finite learning rate is still possible. More specifically, we assume that V is bistable for $Q < Q_c$ and exhibits in this parameter domain two minima at $m = 0$ and $m > 0$. For $Q < Q_c$ the counterpart of Eq. (2) reads

$$m(Q \downarrow) = 0 \quad \text{or} \quad m(Q \downarrow) = f(Q) > 0. \quad (3)$$

Although the state $m = 0$ is available, the motor control system settles down in the state $m > 0$ as a result of the overdamped potential dynamics given by Eq. (1). In doing so, un-supervised learning in post-training periods can be explained as a hysteresis effect.

Let us elucidate our approach by comparing it with a well-known self-organization phenomenon of an equilibrium system: the emergence of ferromagnetism, see Table 3. In the case of ferromagnetism we have the temperature as a control parameter. At high temperatures the ferromagnetic material shows a paramagnetic macroscopic state with vanishing magnetization. At low temperature there is a ferromagnetic state with finite permanent magnetization. In this example the magnetization is the order parameter of the system. The model for differential learning outlined above concerns a nonequilibrium system, see Table 3 (part B). In that case the control parameter is the degree of variation among the training exercises (the noisiness Q). At low noise levels the macroscopic state is assumed not to exhibit any success of learning. In contrast, at higher noise levels the macroscopic state is characterized by a successful differential learning process. The order parameter in our model is the learning rate, which is zero for low noise levels and finite for higher noise levels.

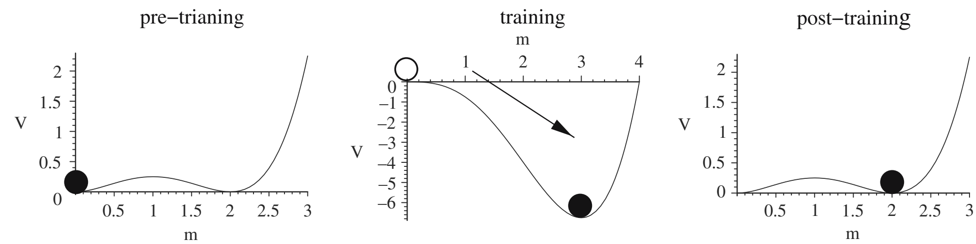

Fig. 4 Adapted fourth-order polynomial potential (4) of differential learning. *Black* and *white balls* represent stable and unstable states, respectively. Parameters (from left to right): $Q - Q_c = -1, 0, -1$. Other parameters: $c = 3$, $d = 1$

These general consideration can be made more explicit by choosing an appropriate potential function $V(m)$.

2.3 Order parameter equation involving a fourth-order polynomial potential

2.3.1 Basics

Potentials that correspond to fourth-order polynomials are frequently used to describe hysteresis phenomena. For example hysteresis effects in ferromagnets and liquid crystals are typically described by means of fourth-order polynomial potentials (Plischke and Bergersen 1994; Frank 2005a, b). The potential involves a bistable parameter domain which is the reason why it is most suitable for our purposes. More explicitly, we consider the potential function

$$V(m) = -\left(\frac{Q - Q_c}{2}\right) m^2 - \frac{c}{3} m^3 + \frac{d}{4} m^4, \tag{4}$$

where c and d are positive constants such that the-order parameter equation (1) becomes

$$\frac{\mathrm{d}}{\mathrm{d}t} m(t) = \frac{1}{\tau_D}\left[(Q - Q_c)m + c\,m^2 - d\,m^3\right]. \tag{5}$$

We see that the potential $V(m)$ has two extrema at $m = 0$ and $m = c/(2d) + \sqrt{c^2/(4d^2) + (Q - Q_c)/d} > 0$ (assuming that the constraint $c^2 > 4d(Q_c - Q)$ holds) corresponding to fixed points of the order parameter equation (5). The potential is bistable for $Q < Q_c$ and monostable for $Q > Q_c$. That is, for $Q < Q_c$ the fixed point $m = 0$ corresponds to a potential minimum and is stable, whereas for $Q > Q_c$ it corresponds to a maximum and is unstable. The fixed point $m > 0$ is stable in any case.

Figure 4 illustrates for the noise-induced transition to self-organized differential learning and the un-supervised learning in post-training periods for the fourth-order polynomial potential (4). The left panel describes the motor control system before training. We have $Q < Q_c$ and the learning rate m equals zero. In the middle panel of Fig. 4 the potential is shown during training with a noise level Q larger than the threshold Q_c. Figure 4 shows that the potential minimum at $m = 0$ becomes a maximum. From Eq. (1) it follows that $m(t)$ converges in this case to the stable fixed point at $m > 0$. After training, the noise level Q is assumed to drop below Q_c. As shown in the right panel of Fig. 4 in this case the order parameter remains fixed at the potential minimum $m > 0$. Furthermore, Fig. 4 illustrates that the inequality $m(Q > Q_c) > m(Q < Q_c)$ holds, which implies that—according to the model—the learning rate during the training sessions is larger than the learning rate in the post-training period. This model feature is in line with experimental evidence, see Table 1.

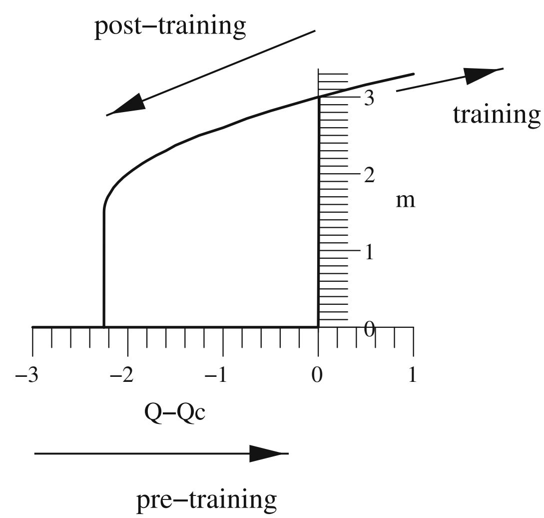

Fig. 5 Hysteresis loop of the adapted fourth-order polynomial potential (4). Parameters c and d as in Fig. 4

The hysteresis loop is illustrated in detail in Fig. 5. Figure 5 shows m as a function of $\Delta Q = Q - Q_c$ computed from Eq. (4). Both the increasing branch ($Q \uparrow$) with the bifurcation at $\Delta Q = 0$ related to the pre-training and training situation and the decreasing branch ($Q \downarrow$) related to the post-training situation are shown.

2.3.2 Application to data

The potential (4) involves four parameters Q, Q_c, c and d. Let us fix c and d and determine $\Delta Q = Q - Q_c$ for the training and post-training stages from the experimental data shown in Table 1 such that the aforementioned constraint $c^2 > 4d(Q_c - Q)$ holds. Using $d = 1\,\text{week}^2/\text{cm}^2$ and $c = 6\,\text{week/cm}$, we find $\Delta Q = 112$ and $\Delta Q = -8$ for the training and post-training period. The corresponding potentials

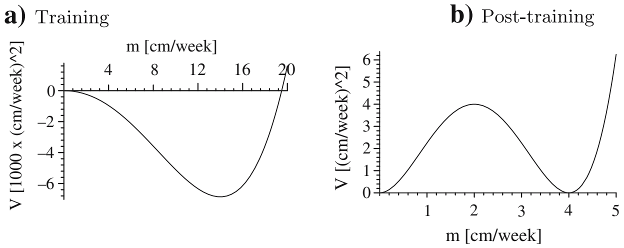

Fig. 6 Illustration of the fourth-order polynomial potential (4) for the shot put data listed in Table 1. **a** Training, **b** Post-training

are shown in Fig. 6 and exhibit minima at $m = 14$ cm/week and $m = 4$ cm/week as listed in Table 1. Note however that the two potential functions differ on a scale of 10^3 (see Sect. 3).

2.4 Predictions: traditional versus differential training

The objective in this section is to find an order parameter description that unifies the traditional and differential training approaches. To this end, we need to reinterpret previous learning studies related to traditional training methods.

As mentioned in the introduction, a dynamical systems perspective of learning has been previously developed in the context of traditional training methods (Schöner 1989). Accordingly, an environmentally specified attractor becomes internalized during motor learning. For example, during learning of bimanual rhythmic finger movements with a phase shift of 90 degrees the environmentally specified relative phase $\psi_{\rm env}$ becomes the center $\psi_{\rm mem}$ of an attractor that emerges during motor learning and represents memorized information. It has been proposed that the $\psi_{\rm mem}$ satisfies $d\psi_{\rm mem}(t)/dt = -\tau_{\rm learn}^{-1} \sin(\psi_{\rm mem} - \psi_{\rm env})$, where $\tau_{\rm learn}$ defines the time scale of significant changes of the "memory" attractor (Schöner 1989). Therefore, the learning rate m is approximately reciprocal to $\tau_{\rm learn}$ such that

$$m \propto \frac{1}{\tau_{\rm learn}}. \tag{6}$$

One caveat: Eq. (6) is a crude simplification of the actual learning process discussed in Schöner (1989). In fact, it has been argued that performance improvement due to motor learning involves several bifurcations (two bifurcations in the study by Schöner 1989) such that the learning rate would become explicitly time-dependent. However, in order to make contact with the experiments discussed in Sect. 2.1 and the order parameter dynamics discussed in Sect. 2.2, we ignore these details and take a broader point of view. We just ask for the expected overall learning rate that can be observed over an extended period of learning. In this case, the theoretical considerations in Schöner (1989) indeed suggest that the learning rate should scale with the typical time scale of the learning process that is defined by the parameter $\tau_{\rm learn}$. Equation (6) can alternatively be expressed as $m = C/\tau_{\rm learn}$, where C is a proportional factor. The ratio $C/\tau_{\rm learn}$ can be related to experimental data (e.g., to the learning rate $m = 4.5$ cm/week of the T group during training see Table 1). In analogy to Eq. (1), we assume that the order parameter $m(t)$ relaxes to the fixed point $C/\tau_{\rm learn}$ such that we have

$$\frac{d}{dt}m = -\frac{1}{\tau_T}\left(m - \frac{C}{\tau_{\rm learn}}\right), \tag{7}$$

where τ_T defines the relaxation time of the order parameter dynamics.

Finally, let us combine the order parameter equations (1) and (7) for differential and traditional learning. To this end, we consider a training group M (where the label M stands for "mixed") that performs exercises according to both training methods. That is, the training program of the M group is composed of p percent differential training sessions and q percent traditional training sessions with $p + q = 1$. In this situation performance improvement due to the internalization of an environmentally specified input can occur only in q percent of the total number of training sessions. Therefore, the effective time scale of learning $\tau_{\rm eff}$ becomes larger than $\tau_{\rm learn}$ by a factor q: $\tau_{\rm eff} = \tau_{\rm learn}/q$. In other words, we obtain a modified steady state learning rate $m = Cq/\tau_{\rm learn}$ such that the order parameter dynamics (7) becomes

$$\frac{d}{dt}m = -\frac{1}{\tau_T}\left(m - C\frac{q}{\tau_{\rm learn}}\right). \tag{8}$$

If the M group performs the two types of training exercises in a randomized fashion, the evolution of m will be determined both by Eqs. (1) and (8). In this case, m will satisfy the order parameter equation

$$\frac{d}{dt}m = -\frac{q}{\tau_T}\left(m - C\frac{q}{\tau_{\rm learn}}\right) - \frac{p}{\tau_D}\frac{dV(m)}{dm} + N(m). \tag{9}$$

Note that above we have weighted Eqs. (1) and (8) with the percentages p and q. Moreover, we have introduced the term $N(m)$ that represents non-additive interference effects between both training methods. Since for $p = 0$ and 1 interference effects can not occur we conclude that the N-term must vanish in these special cases. Consequently, Eq. (9) reduces for $(p, q) = (1, 0)$ and $(p, q) = (0, 1)$ to Eqs. (1) and (8), respectively.

Due to the lack of experimental data, it is difficult to gain further insights into the function $N(m)$. However, Eq. (9) is a powerful tool for predicting the learning rate m in the case of negligible interference effects. The reason for this is that the parameter $C/\tau_{\rm learn}$ and the potential $V(m)$ can be determined from typical studies on differential learning that involve a T group and a D group (cf. Sect. 2.3.2 and see above). Once

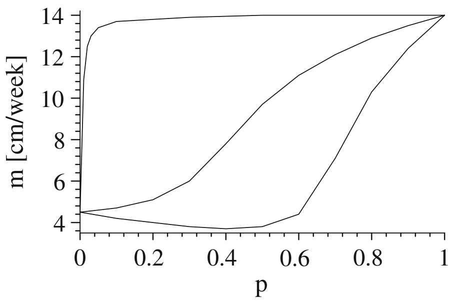

Fig. 7 Predicted learning rate for a hypothetical M group as a function of the percentage p of differential training sessions as computed from Eq. (9). From top to bottom: $\tau_T/\tau_D = 1, 1/100, 1/300$. Other parameters: $C/\tau_{\mathrm{learn}} = 4.5$ cm/week, $c = 6$ cm/week, $d = 1$ (cm/week)^2, $Q - Q_{\mathrm{c}} = 112$

we have determined empirically C/τ_{learn} and $V(m)$, we can compute the steady state value m for all percentages p by solving Eq. (9) numerically. This has been done for the shot put study described in Sect. 2.1 using the data listed in Table 1 and the potential $V(m)$ determined in Sect. 2.3.2. The result is shown in Fig. 7 for different ratios τ_T/τ_D.

3 Summary and discussion

As opposed to the objective in traditional learning methods, the objective in differential learning is to induce a self-organized learning process during which subjects improve their performance in line with their individual and context-dependent abilities and preferences. To this end, subjects are trained by a large variety of exercises. Due to these exercises subjects perform and become aware of the whole range of movement patterns that are potentially available to them.

Under the working hypothesis of self-organized differential learning, in previous experimental works evidence has been found that the differential learning method is superior to traditional learning methods—as far as complex motor skills such performing a shot put or a tennis serve are concerned. In this work we have developed a quantitative dynamical theory of differential learning of complex motor skills.

3.1 Generality

The dynamical model developed in Sect. 2.2 is a general model to the extent that it can be applied not only to the shot put study reviewed in Sect. 2.1 but it can address—at least qualitatively—results obtained in similar differential learning studies concerned with the learning of complex motor skills. That is, we are inclined to say that our model can also be used to describe the emergence of differential learning process during tennis training and volley ball training (Humpert 2004; Humpert and Schöllhorn 2006; Römer et al. 2003).

3.2 Experimental accessibility

Parameter estimation In order to illustrate that our dynamical systems approach can be quantitatively applied to experimental data, we fitted in Sect. 2.3.2 the parameters of the fourth-order polynomial potential (4) to the shot put data listed in Table 1. The potentials $V(m)$ for the training and post-training period are shown in Fig. 6. In order to obtain these potentials, we had to fix the parameters c and d. One of these parameters, say d, could be eliminated by putting it into the time parameter τ_D. Therefore, the question is how to determine the remaining unknown parameter c from experimental data. To this end, however, we can not proceed as in Sect. 2.3.2. In Sect. 2.3.2 we derived the relative noise levels $\Delta Q = Q - Q_{\mathrm{c}}$ from the observed learning rates m given fixed values for c and d. In general, estimates for c, d, and Q_{c} can be obtained from experimental studies that determine the learning rate m for more than one noise level Q. Data from such an experiment given in terms of a function $m(Q)$ could then be fit to the fixed points $m(Q)$ as predicted by the potential (4). Such a fit would yield the unknown parameter c, d, Q_{c}.

Critical phenomena Although our model in Sect. 2.3 involves a random element in terms of the noisiness parameter Q, as such it describes a deterministic evolution equations for the learning rate m. A stochastic generalization of our model can be carried out with relatively little effort. Such a generalized model leads to predictions that can be tested in future experimental studies. For example, such a generalized stochastic model predicts the emergence of critical phenomena (Haken 2004) such as critical fluctuations and critical slowing down close to the onset of the self-organized learning process (bifurcation point). Evidence for critical fluctuations and critical slowing down has indeed be found in studies on human movement (Schöner et al. 1986; Scholz et al. 1987; Kelso 1995; Haken 1996) and movement-related brain activity (Wallenstein et al. 1995). Therefore, there is a good chance that the critical phenomena predictions of a stochastic version of our model could be verified or falsified in appropriately defined experimental studies.

Mixed learning processes In Sect. 2.4 we predicted the learning rate for a mixed training group that is trained by means of traditional and differential training methods. An effective order parameter equation that can be evaluated numerically (see Fig. 7) was proposed for learning situations in which interference effects of these two methods can be neglected. The proposed model involves a single fit parameter given by the ratio τ_T/τ_D that describes the relative impacts of the traditional versus the differential learning dynamics. The model can be verified or falsified experimentally. To this end, the learning rate of several mixed training groups with different

Table 4 Comparison of learning processes from a dynamical systems perspective

Learning process	Type of dynamical process	To-be-learned movement pattern specified by training instructions (environmental input)
Differential learning	Noise-induced self-organized process	No
Repetition	Driven process involving a deterministic driving force	Yes
Schema	Driven process involving a driving force with variations	Yes

p parameters could be determined. The empirically derived function $m(p)$ could then be plotted versus predicted functions as shown in Fig. 7. In doing so, the relevance of interference effects between traditional and differential training methods could also be addressed. In particular, one could address the question to which extent traditional training exercises induce differential learning (i.e., the interference term N is proportional to the gradient of the differential learning potential V) and to which extent differential training exercises induce traditional learning (i.e., the term N is proportional to the linear traditional learning dynamics).

3.3 Differential learning and alternative approaches to learning

Learning with and without a leader According to our modeling approach, differential learning features two key properties. First of all, it is a self-organized process, see Table 4. Second it emerges due to a bifurcation (see also Discussion below and Sect. 3.4). The first key property concerns the relationship between the environmental input (in terms of training instructions) and the produced movement pattern. The movement pattern is not specified by the input. That is to say the input does not force a sports student to produce a particular movement pattern. According to our interpretation as a noise-induced bifurcation, the most relevant ingredient of the training is the variation between the exercises. It is clear that the variation cannot specify a particular movement pattern. The fact that the environmental input is not specific to the movement pattern however is a generic feature of self-organized systems. In the theory of self-organization, it is well-known that the control parameter does not specify the macroscopic pattern of a self-organized system. For example, in the Bénard convection the environmental input is the temperature difference and the macroscopic output is a roll pattern. However, the temperature difference as such does not contain any information about the emergent roll pattern. In contrast in traditional studies on motor skill learning the relationship between the environmental input and the movement pattern is typically a master-slave relationship. In learning studies using repetition of a single movement pattern as training method the environmentally defined training pattern is the movement pattern that is actually learned by the sports student. As mentioned in Sect. 2.4 the dynamical system can be regarded as a driven system for which the environmental input (in terms of training instructions) specifies the macroscopic output. This kind of classification also applies to training studies involving exercise variability in the context of schema theory. A participant in such a training study is typically asked to learn a particular environmentally specified movement class (schema) and the corresponding parameter tuning law. We are inclined to say that we are dealing in this case again with a driven system. The difference between simple single-movement repetition studies and learning studies addressing the variability of practice hypothesis of the schema theory that in the former studies the driving force is deterministic whereas in the latter studies it includes some variations.

Variability in schema theory and differential learning Let us elucidate the notion of variability in the context of schema theory and differential learning by means of an illustrative example. To this end, let us consider a jazz trio that practices to improve its overall performance. The trio may practice the same piece of music in all of their sessions. In doing so, it may vary the tempo of the performance and the individual members of the trio may change the sound volume of their instruments from forte to piano. In sum, the training of the trio may consists of tempo and volume variations of a given piece of music. Alternatively, the trio may just perform collective improvisations. In this case, the three players would not focus on an external, a priori given piece of music but the focus would be on the potentially possible relations between themselves. The members of the jazz trio would experience situations that they would not be able to encounter by performing variations of a given piece of music. In short, the performance variability during collective improvisations is qualitatively different from the variability that corresponds to the variations of a piece of music. Now, let us regard the jazz players as the counterparts of the components of a motor control system. Then training sessions of the first kind could be regarded as training sessions as inspired by schema theory. In contrast, training sessions of the second kind, i.e., the improvisation sessions, could be regarded as counterparts of training sessions used in differential learning studies.

Link to schema theory? Irrespective of these differences between differential learning processes and learning processes considered in schema theory studies, we would like to make a speculative comment on the relationship between the schema theory and our proposed model of self-organized differential learning. In some schema theory studies it was found that training variability increased the success of a learning process (van Rossum 1990; Shea and Wulf 2005). A speculative explanation of this phenomenon would be that in these reported cases the human motor control systems of the participants were pushed by the training variability close to bifurcation points of differential learning processes. From a dynamical systems perspective this implies that the systems lived close to so-called ghost attractors (i.e., attractors that did not exist for the current parameter values but would have existed for other parameter values). In general, systems close to ghost attractors can show properties similar to those of the systems that indeed exhibit such attractors (for an example about ghost attractors related to human tracking movements see Patanarapeelert et al. 2006). Consequently, we speculate that in those reported cases of schema theory studies in which exercise variability resulted in a higher learning success this learning improvement was probably due to the fact that the relevant human movement systems—when regarded as dynamical systems—lived close to ghost attractors that represented differential learning processes.

Dynamical systems approaches to learning of simple and complex motor tasks

Similarities We have pointed out that there is a close link between self-organization and bifurcation theory. In particular, we argued that the working hypothesis of self-organized differential learning implies that differential learning emerges from a bifurcation. This bifurcation is noise-induced. That is, the control parameter corresponds to the variation of the training exercises. Bifurcations as such are a central topic in the dynamical systems approach to human movement (Kelso 1995; Beek et al. 1995; Peper et al. 1995; Peper and Beek 1998; Amazeen et al. 1998; Daffertshofer et al. 1999; Sternad 2000). In these studies “bifurcation” refers to a transition between performance patterns (context A). In our study “bifurcation” refers to a transition to a particular type of learning, namely, differential learning (context B). These two levels of consideration should not be considered as being separated. Taking a structural point of view, learning dynamics (context B) describes the evolution of parameters that occur in low-dimensional dynamical models of motor performance (context A) that in turn describe the emergence of novel performance patterns and bifurcations between performance patterns (Schöner 1989).

Differences Learning of simple motors skills [such as tapping with a particular phase relationship to the beat of a metronome (Kelso 1995) or oscillating two limbs with a particular relative phase (Schöner et al.1992)] typically involves a single key motor variable, e.g., the relative phase. During training participants learn to control this key motor variable such that it approaches a particular required level. In line with the notion of self-organization the training results in a change of the relations between the components of the task-related motor control system. The relations between components converge to a particular fixed set of entities that describe the component-component couplings required for an optimal performance. The behavior of the key motor variable results from the interactions between component as defined by the coupling functions. In turn, the behavior of the key motor variable determines the component behavior (circular causality principle Haken 2004). Therefore, the key motor variable can be regarded as an order parameter.

Learning of complex motor skills such as performing a shot put or a tennis serve does not involve at all a single key motor variable because we are usually dealing with multi-joint movements. The goal of differential training is to establish or improve the ability of a participant to find successful behavioral patterns under permanently changing internal and environmental conditions. Consequently, the relations between the motor control components do not converge to a fixed set of coupling function. The relations are permanently changing. Differential learning is not about the emergence of a particular synergy, principle component or Fourier mode whose amplitude could be identified as order parameter of the motor control system. Therefore, theoretical accounts of differential learning that involve a particular motor variable or a particular amplitude of a principle component mode will fail. In contrast, differential learning is about the emergence of a novel learning behavior. This learning behavior can be regarded as the counterpart to a roll pattern that emerges due to thermal convection. In the pre-transition regime the rotation velocity of the roll pattern equals zero. At the transition point the velocity becomes finite. Likewise, according to the model outlined in Sect. 2 in the pre-transition domain the success of the differential learning equals zero. At the transition point the success becomes finite. In the Bénard convection experiment the order parameter describes the speed at which the roll pattern revolves around.[1] In our model the order parameter describes how fast the performance is improved (i.e., the success or learning rate) as a result of the emergence of the novel differential learning behavior, see Fig. 8.

Limitations The fact that our proposed order parameter (i.e., the learning rate) does not correspond to a motor variable imposes some limitations on differential learning studies. First, the learning rate cannot be observed during training sessions but has to be determined by means of post-tests at the

[1] In fact, the order parameter is a vector that includes the velocity as a component. For details see e.g., Haken (2004).

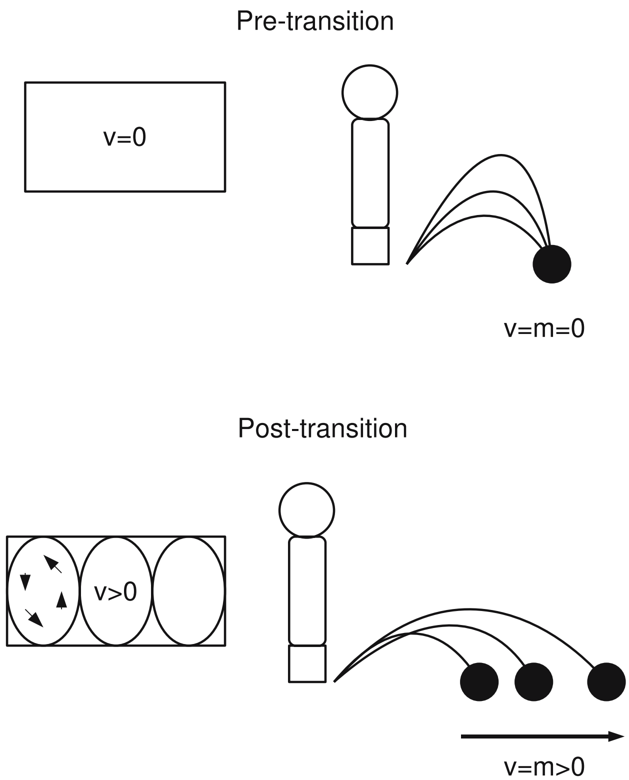

Fig. 8 *Right* Illustration of the proposed order parameter m for differential learning. Learning rate m in the shot put study describes how fast the throwing range is extended. That is, m can be regarded as the "throwing distance increase velocity" v observed in the differential learning training group. *Left* Analogy to the order parameter v (rotation speed) of Bénard convection rolls

end of training periods. That is, in learning studies on simple motor skills described by a single motor variable the learning process can be directly observed during the training sessions by recording the relevant motor variable as a function of learning time. In contrast, in order to observe the evolution of the proposed order parameter for differential learning, we need to interrupt training sessions by scanning sessions. Second, the learning rate cannot be manipulated as easily as a motor variable (e.g., the relative phase). For example in perturbation studies a motor variable can be manipulated by applying an external force to the motor control system. In our case, with the learning rate as order parameter at hand, perturbation studies (e.g., to address the issue of critical slowing down, see below) have to be designed very carefully.

3.4 Conceptual implications

Differential stability of learning attractors during training and post-training periods The potentials derived in Sect. 2.3.2 exhibited minima at the experimentally observed learning rates m. However, the potentials differ on a scale of 10^3. The potential-well during training is much more pronounced than the potential-well associated to the post-training period. In other words, the overall attractor is stronger during training than in the post-training period. The question is whether or not this observation is a generic one. Does it depend on the choice of the parameters c and d? Can we observe a similar phenomenon in other learning studies?

"Activation energy" of differential learning In Sect. 2.1 we reviewed a learning study on shot put in which during post-training periods the performance improved for participants of differential learning groups but not for participants of traditional learning groups. In Sect. 2.2 we interpreted this observation as a hysteresis effect of a potentially bistable system. The bistability as illustrated in Fig. 4 has far-reaching theoretical and practical implications. From a theoretical point of view, our modeling approach suggests that self-organized differential learning as such is accessible to everybody at any time. The noisy training by means of differences is just one means to activate this kind of learning process. In analogy to chemistry, we may consider the potential barrier that is shown in the left panel of Fig. 3 and separates the minima at $m = 0$ and $m > 0$ as an activation energy that is required to trigger the process of self-organized learning. This activation energy is provided by the noisy training sessions. Just as in chemistry, the activation energy for differential learning has to be provided only once. In line with our model, we speculate that the activation can be provided by several means. Noisy training sessions have been turned out to be successful for that purpose. However, it would not come as a surprise if in future studies different activation mechanisms would be discovered. In this context, the question arises why novices are trapped in the state of unsuccessful random learning indicated by the potential minimum at $m = 0$, see Fig. 4. Are we born in this state? Are our learning experiences during early development such that we are pushed into this state?

Un-learning due to a lack of environmental stimuli? The hysteresis loop depicted in Fig. 5 reveals that there is another bifurcation that has not been discussed so far. For small noise levels Q such that $Q - Q_c$ becomes smaller than a critical value the order parameter dynamics becomes monostable with a stable fixed point at $m = 0$. In Fig. 5 this is the case for $Q - Q_c < -2.25$. This implies that an environment that offers experiences with little variations can "destroy" a subject's ability of self-organized differential learning. More precisely, there is a bifurcation of the learning rate from $m > 0$ to $m = 0$ indicating that a subject is set back into the state of an unsuccessful random learning behavior due to a lack of variations. This observation is of particular interest for two reasons. First, we see that we can indeed imagine a mechanism that pushes humans into the state $m = 0$. This is the assumed initial state of all subjects participating in the learning experiments reviewed in Sect. 2.2. This would also

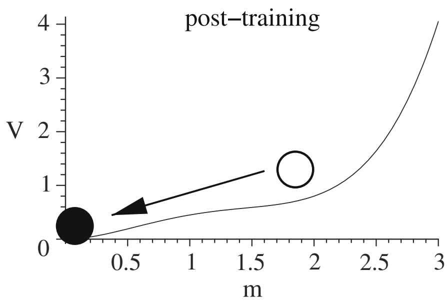

Fig. 9 Adapted fourth-order polynomial potential (4) for small noise levels Q below the second threshold. Parameter: $Q - Q_c = -2.4$. Other parameters as in Fig. 4

provide a partial answer to the question why subjects are initially (i.e., before training) not in the state of differential learning. Second, our model shows that differential learning not necessarily leads to performance improvement during post-training periods. If the noise levels Q in post-training periods is below the aforementioned critical value the attractor minimum of differential learning at $m > 0$ vanishes, see Fig. 9. For example, in a high jump study involving a traditional and differential learning training group a retention test after learning did not show a significant improvement for both groups (Welminski 2005).

Saturation versus re-entrant bifurcations, and the route to chaos by means of higher-order bifurcations In Sect. 2.2 we discussed the general form of the order parameter potential $V(m)$. An explicit example related to a fourth-order polynomial potential was examined in Sect. 2.3. In general, the learning rate $m = f(Q)$ as a function f of the noise level Q of the training exercises will crucially depend on the choice of our model. For example, the fourth-order polynomial model provided us with a continuous range of learning rate values. From Eq. (4) it follows that the larger Q the larger is the steady state learning rate m. Mathematically speaking, in the limiting case $Q \to \infty$ we have $m \to \infty$. This feature of the fourth-order polynomial model might not be a realistic one. It would be more plausible to assume that m converges to a saturation level or that m becomes smaller if the noisiness of the training exercises becomes too large such that the context between exercise and to-be-learned motor skill is lost. In the latter case we would obtain some kind of resonance curve $m(Q)$. In general, more sophisticated order parameter models may be applied. In particular, it would be worth while to consider order parameter equations that describe so-called re-entrant bifurcations (Frank 2005a). Such models would predict that the learning rate decreases towards zero if the noise level becomes too large. Alternatively, we may speculate that—in analogy to the Bénard convection—differential learning could exhibit secondary and higher-order bifurcations when the degree of training noise is increased even further. In line with this consideration, we would expect that the learning rate finally becomes chaotic. As a result of the sensitivity to initial conditions of chaotic systems, we would not be surprised to find that during differential training sessions operating in such a chaotic regime the performance of some participants would dramatically improve while the performance of other participants would not improve at all.

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